The expression for exterior-surface strain factor is obtained from Eq. (12) by substituting $r = b$

$$SF = \frac{\epsilon_{tb} E}{\sigma_y} = \frac{2\rho^2}{\sqrt{(3b^4 + \rho^4)}} \tag{23}$$

Solving Eq. (23) for ρ and substituting in Eq. (22) yields:

$$PF = \frac{1 \cdot 08}{4} \log \left(\frac{3SF^2\, W^4}{4 - SF^2}\right) + \sqrt{\left(\frac{4 - SF^2}{12}\right)} - \frac{SF}{2} \tag{24}$$

The plots of this relationship are shown in Fig. 9 along with the experimental data. As can be seen, very close agreement was obtained between Eq. (24) and the experimental averages.

PERMANENT ENLARGEMENT RATIO

An important factor in the design of thick-wall cylinders using autofrettage is the ratio of the permanent enlargement at the bore to that at the outside

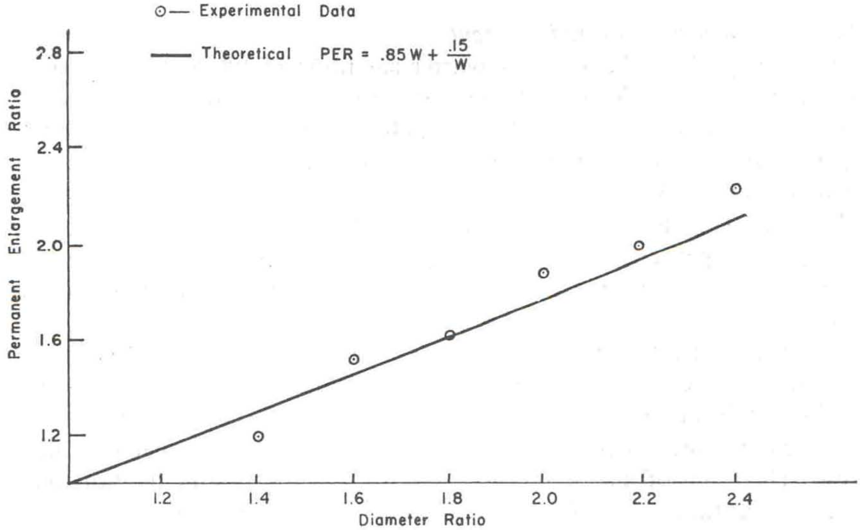

FIG. 10. Permanent enlargement ratio vs. diameter ratio.

surface. This ratio was obtained by physical measurements for all specimens tested. No correlation was found between permanent enlargement ratio and percent bore enlargement and, therefore, all values for the same diameter ratio were average. These averaged values are shown as experimental points in Fig. 10.

The theoretical relationship for permanent enlargement ratio may be derived as follows. The enlargement of the bore under pressure is given by Eq. (21) substituting $r = a$. To find the permanent enlargement of the bore, the elastic recovery must be substracted from the enlargement under pressure. The elastic recovery is given by the Lamé equation and the value of pressure from Eq. (22).

The enlargement of the outside surface under internal pressure is found from Eq. (12), letting $r = b$ and the elastic recovery of the exterior surface is again subtracted as was that for the inside surface. Dividing the resulting equation for permanent bore enlargement by that for permanent enlargement of the outside surface yields the following equation for the permanent enlargement ratio.

$$PER = \frac{1}{2}\left[(2 - \mu)\, W + \frac{\mu}{W}\right] \tag{25}$$

A plot of this equation is shown in Fig. 10 for $\mu = 0{\cdot}3$.

It is interesting to note that Eq. (25) can be derived directly from geometric considerations assuming no net change in volume as a result of overstrain.

Percent Permanent Bore Enlargement

From Fig. 9 it can be seen that, when a condition of 100 percent overstrain (strain factor = 1.0) is reached, the pressure-strain curve becomes essentially flat. It is, therefore, assumed that no additional benefit can be obtained by further deformation and that 100 percent overstrain is the optimum amount of deformation. Actually, in higher diameter ratios where reverse yielding will occur at the bore on the release of the 100 percent overstrain pressure, the optimum percent bore enlargement may be slightly less. However, in the diameter ratios investigated, this reverse yielding effect is considered negligible.

The percent permanent bore enlargement required to produce 100 percent overstrain was determined experimentally by plotting the maximum exterior surface strain factor vs. the percent permanent bore enlargement obtained for each specimen. This yielded a family of curves, one for each diameter ratio. The point of intersection of each of these curves with the horizontal line representing a strain factor of 1.0 indicated the value of permanent bore enlargement to just produce 100 percent overstrain. These points are plotted in Fig. 11.

The theoretical curve shown is obtained by substituting $r = a$ and $\rho = b$ in Eq. (21) yielding the following equation for bore strain at 100 percent overstrain pressure.

$$\epsilon_{tao} = \frac{\sigma_y}{E}\left[-1.08(1 - 2\mu)\ln W + \frac{\mu}{2} + \left(\frac{2 - \mu}{2}\right) W^2\right] \tag{26}$$